Pearson Edexcel

Mark Scheme (Results)

Summer 2021

Pearson Edexcel International Advanced Subsidiary/Advanced Level
In Pure Mathematics P1 (WMA11/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Pearson Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ or ft will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- d… or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper or ag- answer given
- ⎣ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.

- If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. Marks for each question are scored by clicking in the marking grids that appear below each student response on ePEN. The maximum mark allocation for each question/part question(item) is set out in the marking grid and you should allocate a score of ‘0’ or ‘1’ for each mark, or “trait”, as shown:

	0	1
aM		●
aA	●	
bM1		●
bA1	●	
bB	●	
bM2		●
bA2		●

9. Be careful when scoring a response that is either all correct or all incorrect. It is very easy to click down the ‘0’ column when it was meant to be ‘1’ and all correct.

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = ...

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = ...

2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

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Question Number	Scheme	Marks
1(a)	$\left(\frac{dy}{dx}=\right) \ldots x^1+\ldots x^{-\frac{3}{2}}+\ldots x^{-2}$	M1
	$\left(\frac{dy}{dx}=\right) \frac{2}{3}x - 2x^{-\frac{3}{2}} - \frac{8}{3}x^{-2}$	A1A1A1
		(4)
(b)	$\frac{dy}{dx}=\text{"}\frac{2}{3}\text{"}\times 4-\text{"}2\text{"}\times 4^{\text{"}-\frac{3}{2}\text{"}}-\text{"}\frac{8}{3}\text{"}\times 4^{\text{"}-2\text{"}}=\frac{9}{4}$	M1
	$\frac{9}{4}\to -\frac{4}{9}$	M1
	$y-3=\text{"}-\frac{4}{9}\text{"}(x-4)$	dM1
	$4x+9y-43=0$	A1
		(4)
		(8 marks)

(a)

M1 Reduces the power by 1 on any of the following terms

$\ldots x^2 \to \ldots x^1, \ldots x^{-\frac{1}{2}} \to \ldots x^{-\frac{3}{2}}, \ldots x^{-1} \to \ldots x^{-2}$, $5 \to 0$

A1 One of $\frac{2}{3}x$, $-2x^{-\frac{3}{2}}$, $-\frac{8}{3}x^{-2}$ or exact simplified equivalent terms eg $\frac{-2}{\left(\sqrt{x}\right)^3}$ or $-\frac{8}{3x^2}$ or $-2.\dot{6}x^{-2}$

Condone $\frac{2}{3}x^1$ and condone double signs such as $+-2x^{-\frac{3}{2}}$ The terms do not need to be on the same line for this mark.

A1 Two of $\frac{2}{3}x$, $-2x^{-\frac{3}{2}}$, $-\frac{8}{3}x^{-2}$ or exact simplified equivalent terms eg $\frac{-2}{\left(\sqrt{x}\right)^3}$ or $-\frac{8}{3x^2}$.

Condone $\frac{2}{3}x^1$ and condone double signs such as $+-2x^{-\frac{3}{2}}$. The terms do not need to be on the same line for this mark.

A1 $\frac{2}{3}x - 2x^{-\frac{3}{2}} - \frac{8}{3}x^{-2}$ or exact simplified equivalent all on one line eg $\frac{2}{3}x - \frac{2}{\left(\sqrt{x}\right)^3} - \frac{8}{3x^2}$.

Do not allow $\frac{2}{3}x^1 - 2x^{-\frac{3}{2}} - \frac{8}{3}x^{-2}$ or double signs and do not isw in this part (including rounding decimals).

(b)

M1 Substitutes $x = 4$ into their $\frac{dy}{dx}$ to find the numerical gradient of the tangent at P.
Condone slips in their working. They must proceed as far as finding a value.
It may be implied by their answer or embedded in further work such as finding the equation of the perpendicular line. Do not be too concerned by the labelling of their $\frac{dy}{dx}$ (It may even be labelled as $y = \dots$)

M1 For a correct attempt at using $m_N = -\frac{1}{m_T}$ or equivalent to find the gradient of the perpendicular.

dM1 It is for the method of finding a line passing through $(4,3)$ with a changed gradient.
Eg $\frac{9}{4} \to \frac{4}{9}$ would be acceptable as a changed gradient.
Look for $(y-3) = \text{changed } m_T (x-4)$ Both brackets must be correct
Alternatively uses the form $y = mx + c$ AND proceeds as far as $c = \dots$
It is dependent only on the first method mark.

A1 $4x + 9y - 43 = 0$ or exact equivalent with all terms on one side = 0.
Accept $\pm A(4x + 9y - 43 = 0)$ where $A \in \mathbb{N}$

Question Number	Scheme	Marks
2(a)(i)		

	$-a+6a+8+a^2=32 \Rightarrow a^2+5a-24=0$ $(a+8)(a-3)=0$ $a=3$ or $a=-8$ and chooses $a=3$ with reason *	M1 dM1 A1* cso
		(3)
(ii)	$3x^3+26x^2-9x=0 \Rightarrow x(3x^2+26x-9)=0$ $x(3x-1)(x+9)$ $(x=)\ 0, \frac{1}{3}, -9$	M1 A1
		(2)
(b)(i)	$(y=)\ 0$ $y^{\frac{1}{3}}=\text{"}\frac{1}{3}\text{"}$ or $y^{\frac{1}{3}}=\text{"}-9\text{"} \Rightarrow y=...$ (or $(-9)^3=...$ or $\left(\frac{1}{3}\right)^3=...$) $(y=)\ \frac{1}{27},\ -729$	B1 M1 A1
		(3)
(b)(ii)	$9^z=\text{"}\frac{1}{3}\text{"} \to z=...$ $(z=)\ -\frac{1}{2}$ only	M1 A1
		(2)
		(10 marks)

(a)(i)

M1 Substitutes in $x=\pm1, y=32$ and proceeds to a 3TQ in terms of *a* with all terms on one side of the equation. Condone the lack of = 0 and condone slips in their rearrangement.

dM1 Attempts to solve their quadratic equation by either factorising, completing the square or the quadratic formula. They cannot just state the roots. It is dependent on the first method mark. In all cases they must show their working to score this mark so:

- Solving by factorising requires the factorised form of their $a^2+5a-24=0$ to be stated before proceeding to the roots i.e $(a+\text{"}8\text{"})(a-\text{"}3\text{"})\ (=0)$
- Solving by using the quadratic formula requires the values for *a*, *b* and *c* to be stated in the formula before proceeding to the roots. They cannot just state the values of *a*, *b* and *c*.

- Solving by completing the square requires eg $\left(a\pm"\frac{5}{2}"\right)^2 \pm ...$ before rearranging to find the roots.

A1* $a=3$ or $a=-8$ and chooses $a=3$ with a minimal reason. Eg "as a is a positive constant" or "Since $a > 0$", "a cannot be negative".
The final mark cannot be scored without the previous method marks being scored and there cannot be any errors even if missing brackets are recovered. Just crossing out -8 without a reason is A0.

(a)(ii)

M1 Takes out a factor of x from the given cubic with $a=3$ (or divides through by x) and attempts to solve the resulting quadratic equation. You must see at least one intermediate line of working before proceeding to the roots eg the factorised form, values in the quadratic formula or completed square form.

A1 $(x=)\ 0,\ \frac{1}{3},\ -9$ provided M1 has been scored.
Solutions with no working in this part scores 0 marks.

(b)(i)

B1 $(y=)\ 0$

M1 Sets $y^{\frac{1}{3}}$ equal to any of their non-zero solutions from part (a) and attempts to cube their value to find a value for y. You must see at least one stage of working to score this mark.
$(-9)^3 = ...$ on its own scores M1.

A1 $(y=)\ \frac{1}{27}, -729$ and no others other than 0.
Solutions without any working will score a maximum of B1M0A0 in this part.

(ii)

M1 Sets 9^z equal to any of their positive solutions found in part (a) and proceeds to find a value for z. They may write 9^z as 3^{2z} before proceeding to find a value for z.
Alternatively, you may see attempts to link b(i) and b(ii) together:
Eg $y=9^{3z} \Rightarrow \frac{1}{27}=729^z \Rightarrow z=...$ which can score M1.
You must see at least one stage of working to score this mark.
The method may include log statements which is acceptable.

A1 $(z=)\ -\frac{1}{2}$ only (provided M1 has been scored).
Answer stated without working scores 0 marks. Evidence of calculator use will also be A0.

Question Number	Scheme	Marks
3(a)	$\left(2\sqrt{2}\right)^2 = p^2 + q^2 - 2pq\cos 60°$ oe	M1
	$p^2 + q^2 - pq = 8$ *	A1*
		(2)
(b)	$q = p + 2 \Rightarrow 8 = p^2 + ("p+2")^2 - p("p+2")$	M1
	$p^2 + 2p - 4 = 0$ or $q^2 - 2q - 4 = 0$	A1
	$p = \dfrac{-2 \pm \sqrt{2^2 - 4\times 1\times(-4)}}{2}$ or $q = \dfrac{2 \pm \sqrt{(-2)^2 - 4\times 1\times(-4)}}{2}$	M1
	$p = -1 + \sqrt{5}$ or $q = 1 + \sqrt{5}$	B1 (A1 on EPEN)
	$p = -1 + \sqrt{5}$ and $q = 1 + \sqrt{5}$ only	A1cso
		(5)
(c)	$\text{Area} = \dfrac{1}{2} \times ("-1+\sqrt{5}")("1+\sqrt{5}") \times \sin 60°$	M1
	Area = $\sqrt{3}$ (m^2)	A1
		(2)
Alt(a)	Forming a line *BX* which is perpendicular to *AC* where *X* is on the line *AC*. $AX = p\cos 60 = \dfrac{p}{2}$ $BX = \sqrt{p^2 - \left(\dfrac{p}{2}\right)^2} = \dfrac{\sqrt{3}}{2}p$ or $BX = p\sin 60$	
	$\left(\dfrac{\sqrt{3}}{2}p\right)^2 + \left(q - \dfrac{p}{2}\right)^2 = \left(2\sqrt{2}\right)^2$ or $\left(p\sin 60\right)^2 + \left(q - \dfrac{p}{2}\right)^2 = \left(2\sqrt{2}\right)^2$	M1
	$\dfrac{3p^2}{4} + q^2 - pq + \dfrac{p^2}{4} = 8$	
	$p^2 + q^2 - pq = 8$ *	A1*
		(9 marks)

(a)

M1 $\left(2\sqrt{2}\right)^2 = p^2 + q^2 - 2pq\cos 60°$ or $\left(2\sqrt{2}\right)^2 = p^2 + q^2 - 2pq \times \frac{1}{2}$ or $8 = p^2 + q^2 - 2pq\cos 60°$

They may carry this out in two stages by forming two right angled triangles with BX being perpendicular to AC (see Alt(a)). To score this mark they must proceed as far as

$\left(\frac{\sqrt{3}}{2}p\right)^2 + \left(q - \frac{p}{2}\right)^2 = \left(2\sqrt{2}\right)^2$ or $(p\sin 60)^2 + \left(q - \frac{p}{2}\right)^2 = \left(2\sqrt{2}\right)^2$

Condone missing brackets for M1.

A1* Achieves $p^2 + q^2 - pq = 8$ with no errors including omission of brackets. One of the lines above must have been seen for M1A1. If they state = 8 without showing any working then A1 cannot be scored.

(b)

M1 Substitutes $q = p + 2$ (oe) into the given equation

A1 $p^2 + 2p - 4 = 0$ or the equivalent equation in q ($q^2 - 2q - 4 = 0$)

M1 Attempts to solve to find p using the formula or completing the square using their values. Alternatively, they achieve a quadratic equation in q and attempt to solve to find q using their values. Usual rules for solving quadratics apply. If they state the roots or factorise then M0. If they use the quadratic formula then the values must be embedded.

B1 $p = -1 + \sqrt{5}$ **or** $q = 1 + \sqrt{5}$ (ignore any other solutions) Must be exact. This is independent of the previous method mark so if the roots are just stated this mark can be scored. (Note this is A1 on EPEN)

A1 $p = -1 + \sqrt{5}$ **and** $q = 1 + \sqrt{5}$ only cso (all other marks must have been scored to award A1)

(c)

M1 Attempts to find the area of the triangle using $\frac{1}{2} \times ("-1+\sqrt{5}")("1+\sqrt{5}") \times \sin 60°$. Must see at least one stage of working using their p and their q

A1 $\sqrt{3}$ (m^2) condone lack of units. Do not accept rounded answers.

Question Number	Scheme	Marks
4	$\int \frac{3x^{\frac{3}{2}}-15x^{\frac{1}{2}}+2x-10}{4\sqrt{x}}\,dx = \int \frac{3}{4}x-\frac{15}{4}+\frac{1}{2}x^{\frac{1}{2}}-\frac{5}{2}x^{-\frac{1}{2}}\,dx$	M1A1A1
	$x^n \to x^{n+1}$	dM1
	$\frac{3}{8}x^2-\frac{15}{4}x+\frac{1}{3}x^{\frac{3}{2}}-5x^{\frac{1}{2}}+C$	A1A1
		(6 marks)

M1 Attempts to write as a sum of terms using correct index laws at least once.
Award for any term with a correct index.
Score for any one of:
$\frac{"3x^{\frac{3}{2}}"}{4\sqrt{x}} \to ...x \quad \frac{"-15x^{\frac{1}{2}}"}{4\sqrt{x}} \to \text{const} \quad \frac{+"2x"}{4\sqrt{x}} \to ...x^{\frac{1}{2}} \quad \frac{"-10"}{4\sqrt{x}} \to ...x^{-\frac{1}{2}}$
Cannot be scored for a term with a correct index arising from incorrect work
eg $\frac{"-10"}{4\sqrt{x}} \to ...x^{\frac{1}{2}}$

A1 Two correct terms of $\frac{3}{4}x \quad -\frac{15}{4} \quad +\frac{1}{2}x^{\frac{1}{2}} \quad -\frac{5}{2}x^{-\frac{1}{2}}$ (oe). They do not need to be seen on the same line for this mark and they do not need to be simplified. Eg $\frac{3}{4}x^1$, $\frac{-10x^{-\frac{1}{2}}}{4}$ are acceptable.

Indices must be processed though. Eg $\frac{3x^{\frac{3}{2}}}{4\sqrt{x}} \to \frac{3}{4}x^{1.5-0.5}$ is not acceptable.

A1 All four correct terms of $\frac{3}{4}x-\frac{15}{4}+\frac{1}{2}x^{\frac{1}{2}}-\frac{5}{2}x^{-\frac{1}{2}}$ (oe) Terms do not need to be simplified for this mark and do not need to be on one line.

dM1 Increases the power of any of their terms by 1. ($x^n \to x^{n+1}$). It is dependent on the first method mark.

A1 Any two terms correct unsimplified (see below) but the indices must be processed. Condone $-\frac{15}{4}x^1$ as a correct term for this mark only.

A1 $\frac{3}{8}x^2-\frac{15}{4}x+\frac{1}{3}x^{\frac{3}{2}}-5x^{\frac{1}{2}}+C$ o.e. all on one line (including the constant C and all simplified).

Do not accept $-\frac{15}{4}x^1$ as a correct simplified term.

Accept alternative correct simplified forms such as $\frac{3}{8}x^2-\frac{15}{4}x+\frac{1}{3}x^{\frac{3}{2}}-5\sqrt{x}+C$ or
$\frac{3}{8}x(x-10)+\frac{1}{3}x^{\frac{1}{2}}(x-15)+C$

Question Number	Scheme	Marks
5(a)	$P_B - P_A = 44.2 - \left(53 - 0.4\times 8^2\right) = ...$	M1
	awrt (£) 16.8 million	A1
		(2)
(b)	(£) 53 (million)	B1
		(1)
(c)	$-1.6t + 44.2 = 53 - 0.4(t-8)^2$	M1
	$\Rightarrow 0.4t^2 - 8t + \frac{84}{5} = 0 \Rightarrow t = ...$	M1
	$t = 10 - \sqrt{58} =$ awrt 2.38 (years)	A1
	$"2.38" < t \ (\leqslant 15)$	A1ft
		(4)
(d)	"The **share value** would be negative" / "the **model** is known to hold for 15 years only (and 20 years is more than 15)."	B1
		(1)
		(8 marks)

(a)

M1 Substitutes $t = 0$ into the equation for P_A and finds the difference between 44.2 and P_A.

A1 awrt (£) 16.8 million.

(b)

B1 (£) 53 million. Condone 53

(c)

M1 Sets $-1.6t + 44.2 = 53 - 0.4(t-8)^2$ They may have already attempted to multiply out $(t-8)^2$ so condone errors that may occur before setting $P_A = P_B$. Also may score for use of any inequality sign.

M1 Rearranges the equation to form a 3TQ on one side of the equation (= 0) or inequality and attempts to solve the quadratic. Condone slips in their working. See general rules for solving a quadratic. They may use a calculator to state the root(s) which is acceptable.

A1 awrt 2.38 (years) o.e. isw after a correct answer

A1ft $"2.38" < t \ (\leqslant 15)$ oe from correct working. You may follow through on their value of t provided $t < 15$. Ignore any reference to an upper limit provided it is $\geqslant 15$

You may see eg $10-\sqrt{58} < t < 10+\sqrt{58}$ which can score this mark.

(d)

B1 "The **share value (or *P*)** would be negative" or "the **model** is known to hold for 15 years only (and 20 is greater than 15)" or equivalent explanations.
Eg "*P* must be greater than (or equal to) zero".

Note "$-4.6 < 0$" is insufficient (it does not refer to the **model** or the **share value** of ***P***)

Question Number	Scheme	Marks
6(a)	$\mathrm{f}'(8)=\dfrac{32}{3\times 8^2}+3-2\sqrt[3]{8}\quad\left(=-\dfrac{5}{6}\right)$	M1
	$y-2="-\dfrac{5}{6}"(x-8)$	dM1
	$y=-\dfrac{5}{6}x+\dfrac{26}{3}$	A1
		(3)
(b)	$\mathrm{f}'(x)=\dfrac{32}{3x^2}+3-2\sqrt[3]{x}=\ldots x^{-2}+3+\ldots x^{\frac{1}{3}}$	
	$x^{-2}\to x^{-1},\ 3\to 3x,\ x^{\frac{1}{3}}\to x^{\frac{4}{3}}$	M1
	$\mathrm{f}(x)=\int\dfrac{32}{3}x^{-2}+3-2x^{\frac{1}{3}}\mathrm{d}x=-\dfrac{32}{3}x^{-1}+3x-\dfrac{3}{2}x^{\frac{4}{3}}+c$	A1A1
	$2=-\dfrac{32}{3}\times 8^{-1}+3\times 8-\dfrac{3}{2}\times 8^{\frac{4}{3}}+c\Rightarrow c=\ldots$	dM1
	$(\mathrm{f}(x)=)\ -\dfrac{32}{3}x^{-1}+3x-\dfrac{3}{2}x^{\frac{4}{3}}+\dfrac{10}{3}$	A1
		(5)
		(8 marks)

Ignore labelling of parts (a) and (b)

(a)

M1 Substitutes $x=8$ to find a value for $\mathrm{f}'(8)$. Condone slips in their substitution and $-\dfrac{5}{6}$ seen will imply this mark.

dM1 It is for the method of finding a line passing though $(8,2)$ using their value for $\mathrm{f}'(8)$. Score for $(y-2)="-\dfrac{5}{6}"(x-8)$ with both brackets correct. If they use $y=mx+c$ they must proceed as far as $c=\ldots$ It is dependent on the previous method mark.

A1 $y=-\dfrac{5}{6}x+\dfrac{26}{3}$

(b)

M1 Integrates by raising the power on one of the terms (ie $x^{-2} \to x^{-1}, 3 \to 3x, x^{\frac{1}{3}} \to x^{\frac{4}{3}}$)

A1 Two terms correct of $-\frac{32}{3}x^{-1}$, $+3x$ or $-\frac{3}{2}x^{\frac{4}{3}}$ seen (or unsimplified equivalents). The indices must be processed.

A1 $-\frac{32}{3}x^{-1}+3x-\frac{3}{2}x^{\frac{4}{3}}(+c)$ seen or unsimplified equivalent. Condone the lack of $+c$ for this mark. $-10.7x^{-1}$ is not a correct term but allow $-10.\dot{6}x^{-1}$.

dM1 Substitutes $x=8, y=2$ into their f(x) and proceeds to find c. It is dependent on the previous method mark and condone slips in their rearrangement to find c.

A1 $(\text{f}(x)=)\ -\frac{32}{3}x^{-1}+3x-\frac{3}{2}x^{\frac{4}{3}}+\frac{10}{3}$ or simplified equivalent. isw after a correct answer

Eg $-\frac{32}{3x}+3x-\frac{3}{2}x^{\frac{4}{3}}+\frac{10}{3}$ or $-\frac{10.\dot{6}}{x}+3x-1.5x^{\frac{4}{3}}+3.\dot{3}$ but do not accept rounded decimals for the coefficients.

Question Number	Scheme	Marks
7(a)	y coordinate $=12$	B1
		(1)
(b)	Gradient of $l_1 = -\frac{3}{4}$	B1
	$\Rightarrow$ Gradient of $l_2 = \frac{4}{3} \Rightarrow (y-6) = "\frac{4}{3}"(x-8)$	M1
	y coordinate $= -\frac{14}{3}$ *	A1* cso
		(3)
(c)	Radius $="12"+\frac{14}{3}=\frac{50}{3}$	B1ft
	Length of arc $="\frac{50}{3}"\times 1.8 = 30$	M1A1cao
		(3)
(d)	Area of sector $= \frac{1}{2}\times("\frac{50}{3}")^2\times 1.8 \quad (=250)$	M1
	$"250"+\frac{1}{2}\times"\frac{50}{3}"\times 8 = "250"+"\frac{200}{3}"$	M1
	$=\frac{950}{3}$ (units^2)	A1cao
		(3)
		(10 marks)

Mark all parts together. May work in degrees.

(a)

B1 12 (Check by the question and also on the diagram). If there is a contradiction then their answer in the main solution takes precedence.

(b)

B1 States gradient of l_1 is $-\frac{3}{4}$ but can be implied by further work. Eg sight of a gradient of $\frac{4}{3}$ in their equation for l_2 can also score this mark.
The value must be identified or used so it cannot just be awarded from a rearranged equation for l_1. Circling the coefficient is acceptable but stating $-\frac{3}{4}x$ with no further work is B0.

M1 Attempts to find the gradient of the perpendicular line $"-\frac{3}{4}" \to \frac{4}{3}$ and attempts to find the equation of l_2. Look for $(y-6)="\frac{4}{3}"(x-8)$ with both of the brackets correct. If they attempt using $y=mx+c$ then they must proceed as far as $c=...$

A1* $-\frac{14}{3}$ cso must be clearly stated as the y coordinate with no errors seen after achieving a correct equation for l_2.

(c)

B1ft Finds the radius of the circle following through on their answer to (a). $"12"+\frac{14}{3}$ is acceptable for this mark or it may be implied by their length of the arc. May be seen on the diagram or in other parts.

M1 Attempts to find the length of the arc with $\theta=1.8$ and their $r="12"+\frac{14}{3}$

A1 30 cao

(d)

M1 Attempts to find the area of the sector with $\theta=1.8$ and their $r="12"+\frac{14}{3}$

M1 **Adds the area of their sector with a correct method to find the area of the triangle**.

There are various ways to find the area of the triangle. They may find the lengths CD and DE using Pythagoras and proceed to find the area of the triangle:

Eg $CD=\sqrt{6^2+8^2}=10$ and $DE=\sqrt{8^2+\left(\frac{32}{3}\right)^2}=\frac{40}{3} \Rightarrow \text{Area}=\frac{1}{2}\times 10\times\frac{40}{3}=\frac{200}{3}$

Alternatively, via the shoelace method:

Eg $\frac{1}{2}\begin{vmatrix} 0 & 12 \\ 8 & 6 \\ 0 & -\frac{14}{3} \\ 0 & 12 \end{vmatrix}=\frac{1}{2}\times\left|\left(8\times-\frac{14}{3}\right)-(12\times 8)\right|=\frac{200}{3}$

A1 $\frac{950}{3}$ cao (accept $316\frac{2}{3}$ or $316.\dot{6}$ but not 316.7)

Question Number	Scheme	Marks
8(a)	$3x^2+6x+9=3(x\pm...)^2\pm...\quad a=3$	B1
	$3x^2+6x+9=3(x+1)^2\pm...\quad a=3\ \&\ b=1$	M1
	$3x^2+6x+9=3(x+1)^2+6$	A1
		(3)
(b)	$(-1,\ 6)$	B1ft
		(1)
(c)	$y=\alpha(x+4)(x+2)(x-3)$	B1
	$6=\alpha(-1+4)(-1+2)(-1-3)$	M1
	$\alpha=-\frac{1}{2}$	A1
	$y="-\frac{1}{2}"(x+4)(x+2)(x-3)\Rightarrow y=...x^3+...x^2+...x+...$	M1
	$A=-\frac{1}{2},\ B=-\frac{3}{2},\ C=5,\ D=12$	A1
		(5)
Alt (c)	$-64A+16B-4C+D=0$	
	$-8A+4B-2C+D=0$	B1
	$27A+9B+3C+D=0$	
	$-A+B-C+D=6$	M1
	One of $A=-\frac{1}{2},\ B=-\frac{3}{2},\ C=5,\ D=12$	A1
	Fully solves their simultaneous equations	M1
	$A=-\frac{1}{2},\ B=-\frac{3}{2},\ C=5,\ D=12$	A1
		(9 marks)

(a)

B1 Achieves $3x^2+6x+9=3(x\pm...)^2\pm...$ or states that $a=3$

M1 Deals correctly with the first two terms of $3x^2+6x+9$
Scored for $3x^2+6x+9=3(x+1)^2\pm...$ or states that $a=3$ & $b=1$

A1 $3x^2+6x+9=3(x+1)^2+6$

This may be done by equating coefficients using the expanded form
$a(x+b)^2+c=ax^2+2abx+ab^2+c$

(b)

B1ft $(-1,6)$ or follow through their $(-b,c)$ from (a). Condone lack of brackets and accept eg $x=-1, y=6$

(c)

B1 Identifies that three factors of the cubic equation are $(x+4)(x+2)(x-3)$

M1 A correct method to find the scale factor by using the minimum point found in part (c). Look for the minimum point to be substituted into their equation for the cubic

A1 Scale factor = $-\frac{1}{2}$

M1 Attempts to multiply $(x\pm4)(x\pm2)(x\pm3)$ to achieve $x^3+........\pm24$. This may have been multiplied by their scale factor so look for $\alpha(x^3+........\pm24)$

A1 $A=-\frac{1}{2},\ B=-\frac{3}{2},\ C=5,\ D=12$. Accept $y=-\frac{1}{2}x^3-\frac{3}{2}x^2+5x+12$

In the alternative method using simultaneous equations

B1 Three correct equations formed using $x=\ -4,-2$ and 3 with $y=0$ in each case.

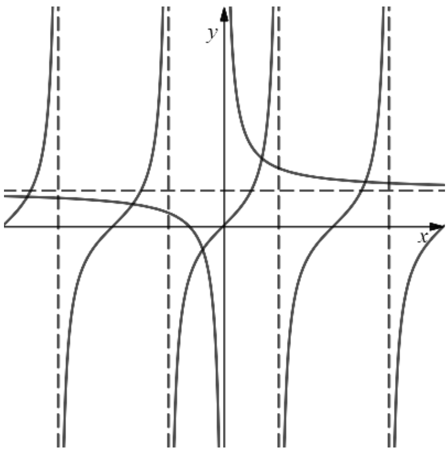
M1 Forms four simultaneous equations using the three intercepts and their point P

A1 One of $A=-\frac{1}{2},\ B=-\frac{3}{2},\ C=5,\ D=12$

M1 Fully solves their simultaneous equations either using matrices or elimination. This may be done on a calculator which is acceptable.

Eg. Using matrices $\begin{pmatrix}-64 & 16 & -4 & 1\\ -8 & 4 & -2 & 1\\ 27 & 9 & 3 & 1\\ -1 & 1 & -1 & 1\end{pmatrix}^{-1}\begin{pmatrix}0\\0\\0\\6\end{pmatrix}=\begin{pmatrix}A\\B\\C\\D\end{pmatrix}\Rightarrow\begin{pmatrix}A\\B\\C\\D\end{pmatrix}=...$

A1 $A=-\frac{1}{2},\ B=-\frac{3}{2},\ C=5,\ D=12$. Accept $y=-\frac{1}{2}x^3-\frac{3}{2}x^2+5x+12$

Question Number	Scheme	Marks
9(a)	$x = \frac{3\pi}{2}$ oe	B1
		(1)
(b)(i)		B1B1
(ii)	5 (solutions)	B1
	Number of solutions are the number of points of intersections between the graphs	B1
		(4)
(c) **(i)**	(Number of solutions) $= 40$	B1ft
(ii)	(Number of solutions) $= 14$	B1
		(2)
		(7 marks)

(a)

B1 $x = \frac{3\pi}{2}$ oe and no others. Do not accept in degrees. It may be labelled on the graph, but it must be an equation. If multiple answers are given then $x = \frac{3\pi}{2}$ oe must be identified (eg may be circled)

(b)(i)

B1 For the shape of a $\frac{1}{x}$ type curve in Quadrant 1. It must not cross either axis and have acceptable curvature – do not penalise candidates unless it is clear that a minimum point was intended.

B1 Correct shape and position for both branches with an asymptote in the correct position **and** labelled as $y = 1$ or stated in their work. Again, do not penalise the sketch unless it is clear that turning points are intended. The asymptote line/dashed line does not need to be drawn on the sketch.

(ii)

B1 5 only

B1 Number of solutions are the number of points of intersections between the graphs. (Do not allow if they mention where they cross the *x*-axis).

(c)

(i)

B1ft 40 Follow through from their sketch
(eg number of intersections in first quadrant $\times 20$)

(ii)

B1 14

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